Chapter 2
An Introduction to Linear Programming

Case Problem 1: Workload Balancing

1.

	Production Rate (minutes per printer)		
Model	Line 1	Line 2	Profit Contribution ($)
DI-910	3	4	42
DI-950	6	2	87

Capacity: 8 hours $\times$ 60 minutes/hour = 480 minutes per day

Let D_1 = number of units of the DI-910 produced
D_2 = number of units of the DI-950 produced

$$
\begin{array}{llllll}
\text{Max} & 42D_1 + 87D_2 & & & \\
\text{s.t.} & & & & \\
& 3D_1 + 6D_2 & \leq & 480 & \text{Line 1 Capacity} \\
& 4D_1 + 2D_2 & \leq & 480 & \text{Line 2 Capacity} \\
& D_1, D_2 \geq 0 & & &
\end{array}
$$

Using *The Management Scientist*, the optimal solution is $D_1 = 0$, $D_2 = 80$. The value of the optimal solution is $6960.

Management would not implement this solution because no units of the DI-910 would be produced.

2. Adding the constraint $D_1 \geq D_2$ and resolving the linear program results in the optimal solution D_1 = 53.333, D_2 = 53.333. The value of the optimal solution is $6880.

3. Time spent on Line 1: 3(53.333) + 6(53.333) = 480 minutes

Time spent on Line 2: 4(53.333) + 2(53.333) = 320 minutes

Thus, the solution does not balance the total time spent on Line 1 and the total time spent on Line 2. This might be a concern to management if no other work assignments were available for the employees on Line 2.

4. Let T_1 = total time spent on Line 1
T_2 = total time spent on Line 2

Whatever the value of T_2 is,

$$T_1 \leq T_2 + 30$$
$$T_1 \geq T_2 - 30$$

Thus, with $T_1 = 3D_1 + 6D_2$ and $T_2 = 4D_1 + 2D_2$

$$3D_1 + 6D_2 \leq 4D_1 + 2D_2 + 30$$
$$3D_1 + 6D_2 \geq 4D_1 + 2D_2 - 30$$

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Hence,

$$-1D_1 + 4D_2 \leq 30$$
$$-1D_1 + 4D_2 \geq -30$$

Rewriting the second constraint by multiplying both sides by -1, we obtain

$$-1D_1 + 4D_2 \leq 30$$
$$1D_1 - 4D_2 \leq 30$$

Adding these two constraints to the linear program formulated in part (2) and resolving using *The Management Scientist*, we obtain the optimal solution $D_1 = 96.667$, $D_2 = 31.667$. The value of the optimal solution is $6815. Line 1 is scheduled for 480 minutes and Line 2 for 450 minutes. The effect of workload balancing is to reduce the total contribution to profit by $6880 - $6815 = $65 per shift.

5. The optimal solution is $D_1 = 106.667$, $D_2 = 26.667$. The total profit contribution is

$$42(106.667) + 87(26.667) = \$6800$$

Comparing the solutions to part (4) and part (5), maximizing the number of printers produced (106.667 + 26.667 = 133.33) has increased the production by 133.33 - (96.667 + 31.667) = 5 printers but has reduced profit contribution by $6815 - $6800 = $15. But, this solution results in perfect workload balancing because the total time spent on each line is 480 minutes.

Case Problem 2: Production Strategy

1. Let $BP100$ = the number of BodyPlus 100 machines produced
$BP200$ = the number of BodyPlus 200 machines produced

Max	$371BP100$	+	$461BP200$			
s.t.						
	$8BP100$	+	$12BP200$	$\leq$	600	Machining and Welding
	$5BP100$	+	$10BP200$	$\leq$	450	Painting and Finishing
	$2BP100$	+	$2BP200$	$\leq$	140	Assembly, Test, and Packaging
	$-0.25BP100$	+	$0.75BP200$	$\geq$	0	BodyPlus 200 Requirement

$$BP100, BP200 \geq 0$$

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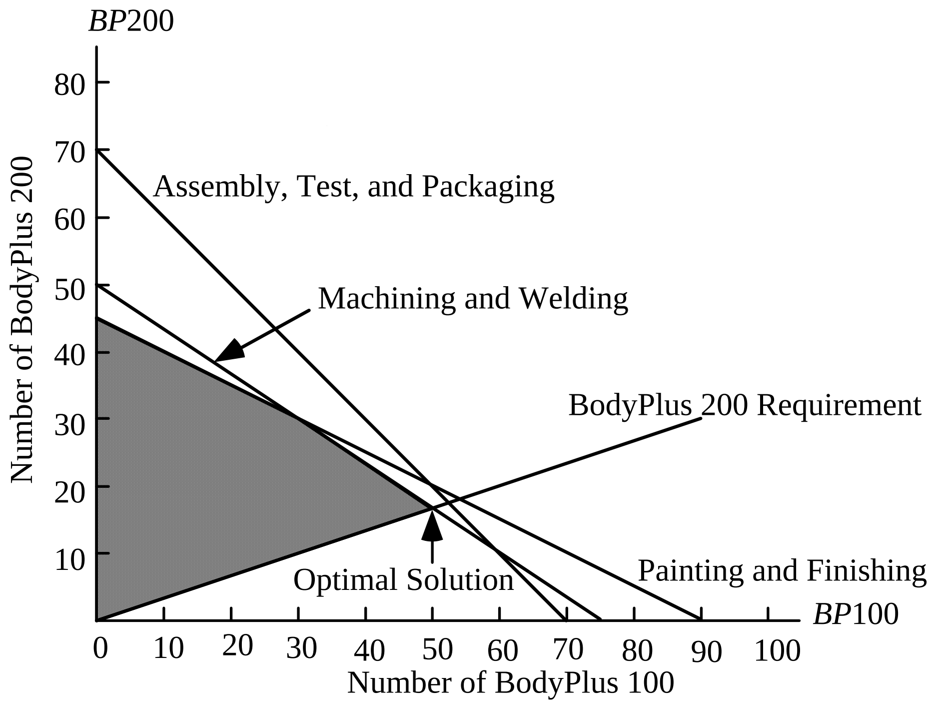

Optimal solution: *BP*100 = 50, *BP*200 = 50/3, profit = $26,233.33. Note: If the optimal solution is rounded to *BP*100 = 50, *BP*200 = 16.67, the value of the optimal solution will differ from the value shown. The value we show for the optimal solution is the same as the value that will be obtained if the problem is solved using a linear programming software package such as The Management Scientist.

2. In the short run the requirement reduces profits. For instance, if the requirement were reduced to at least 24% of total production, the new optimal solution is *BP*100 = 1425/28, *BP*200 = 225/14, with a total profit of $26,290.18; thus, total profits would increase by $56.85. Note: If the optimal solution is rounded to *BP*100 = 50.89, *BP*200 = 16.07, the value of the optimal solution will differ from the value shown. The value we show for the optimal solution is the same as the value that will be obtained if the problem is solved using a linear programming software package such as The Management Scientist.

3. If management really believes that the BodyPlus 200 can help position BFI as one of the leader's in high-end exercise equipment, the constraint requiring that the number of units of the BodyPlus 200 produced be at least 25% of total production should not be changed. Since the optimal solution uses all of the available machining and welding time, management should try to obtain additional hours of this resource.

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Case Problem 3: Hart Venture Capital

1. Let S = fraction of the Security Systems project funded by HVC
   M = fraction of the Market Analysis project funded by HVC

$$\begin{array}{llcllll}
\text{Max} & 1{,}800{,}000S & + & 1{,}600{,}000M & & & \\
\text{s.t.} & & & & & & \\
 & 600{,}000S & + & 500{,}000M & \leq & 800{,}000 & \text{Year 1} \\
 & 600{,}000S & + & 350{,}000M & \leq & 700{,}000 & \text{Year 2} \\
 & 250{,}000S & + & 400{,}000\text{M} & \leq & 500{,}000 & \text{Year 3} \\
 & S & & & \leq & 1 & \text{Maximum for } S \\
 & & & M & \leq & 1 & \text{Maximum for } M \\
 & S,M & \geq & 0 & & &
\end{array}$$

The solution obtained using The Management Scientist software package is shown below:

```
OPTIMAL SOLUTION

Objective Function Value =     2486956.522

      Variable              Value             Reduced Costs
   --------------      ---------------     ------------------
         S                      0.609                   0.000
         M                      0.870                   0.000


     Constraint         Slack/Surplus          Dual Prices
   --------------      ---------------     ------------------
         1                      0.000                   2.783
         2                  30434.783                   0.000
         3                      0.000                   0.522
         4                      0.391                   0.000
         5                      0.130                   0.000


OBJECTIVE COEFFICIENT RANGES

   Variable       Lower Limit       Current Value     Upper Limit
 ------------   ---------------    ---------------  ---------------
       S         No Lower Limit      1800000.000    No Upper Limit
       M         No Lower Limit      1600000.000    No Upper Limit


RIGHT HAND SIDE RANGES

  Constraint      Lower Limit       Current Value     Upper Limit
 ------------   ---------------    ---------------  ---------------
        1        No Lower Limit       800000.000       822950.820
        2            669565.217       700000.000    No Upper Limit
        3            461111.111       500000.000    No Upper Limit
        4                 0.609            1.000    No Upper Limit
        5                 0.870            1.000    No Upper Limit
```


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Thus, the optimal solution is S = 0.609 and M = 0.870. In other words, approximately 61% of the Security Systems project should be funded by HVC and 87% of the Market Analysis project should be funded by HVC.

The net present value of the investment is approximately $2,486,957.

2.

	Year 1	Year 2	Year 3
Security Systems	$365,400	$365,400	$152,250
Market Analysis	$435,000	$304,500	$348,000
Total	$800,400	$669,900	$500,250

Note: The totals for Year 1 and Year 3 are greater than the amounts available. The reason for this is that rounded values for the decision variables were used to compute the amount required in each year. To see why this situation occurs here, first note that each of the problem coefficients is an integer value. Thus, by default, when The Management Scientist prints the optimal solution, values of the decision variables are rounded and printed with three decimal places. To increase the number of decimal places shown in the output, one or more of the problem coefficients can be entered with additional digits to the right of the decimal point. For instance, if we enter the coefficient of 1 for S in constraint 4 as 1.000000 and resolve the problem, the new optimal values for S and D will be rounded and printed with six decimal places. If we use the new values in the computation of the amount required in each year, the differences observed for year 1 and year 3 will be much smaller than we obtained using the values of S = 0.609 and M = 0.870.

3. If up to $900,000 is available in year 1 we obtain a new optimal solution with S = 0.689 and M = 0.820. In other words, approximately 69% of the Security Systems project should be funded by HVC and 82% of the Market Analysis project should be funded by HVC.

The net present value of the investment is approximately $2,550,820.
The solution obtained using The Management Scientist software package follows:

```
OPTIMAL SOLUTION

Objective Function Value =     2550819.672

      Variable           Value           Reduced Costs
   --------------   ---------------   ------------------
         S                   0.689                 0.000
         M                   0.820                 0.000
     Constraint      Slack/Surplus         Dual Prices
   --------------   ---------------   ------------------
         1               77049.180                 0.000
         2                   0.000                 2.098
         3                   0.000                 2.164
         4                   0.311                 0.000
         5                   0.180                 0.000

OBJECTIVE COEFFICIENT RANGES

   Variable       Lower Limit      Current Value     Upper Limit
 ------------   ---------------   ---------------  ---------------
      S         No Lower Limit       1800000.000   No Upper Limit
      M         No Lower Limit       1600000.000   No Upper Limit
```


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```
RIGHT HAND SIDE RANGES

  Constraint      Lower Limit       Current Value     Upper Limit
 ------------   ---------------   ---------------  ---------------
       1             822950.820        900000.000   No Upper Limit
       2         No Lower Limit        700000.000       802173.913
       3         No Lower Limit        500000.000       630555.556
       4                  0.689             1.000   No Upper Limit
       5                  0.820             1.000   No Upper Limit
```

4. If an additional $100,000 is made available, the allocation plan would change as follows:

	Year 1	**Year 2**	**Year 3**
Security Systems	$413,400	$413,400	$172,250
Market Analysis	$410,000	$287,000	$328,000
Total	$823,400	$700,400	$500,250

5. Having additional funds available in year 1 will increase the total net present value. The value of the objective function increases from $2,486,957 to $2,550,820, a difference of $63,863. But, since the allocation plan shows that $823,400 is required in year 1, only $23,400 of the additional $100,00 is required. We can also determine this by looking at the slack variable for constraint 1 in the new solution. This value, 77049.180, shows that at the optimal solution approximately $77,049 of the $900,000 available is not used. Thus, the amount of funds required in year 1 is $900,000 - $77,049 = $822,951. In other words, only $22,951 of the additional $100,000 is required. The differences between the two values, $23,400 and $22,951, is simply due to the fact that the value of $23,400 was computed using rounded values for the decision variables. The value of $22,951 is computed internally in The Management Scientist output and is not subject to this rounding. Thus, the most accurate value is $22,951.

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